Baryon spectroscopy from lattice QCD

- Goal: Determine the hadron mass spectrum of QCD
- New feature: Spin identification for N^* and Δ states
  - R. G. Edwards, J. J. Dudek, D. G. Richards and S. J. Wallace, [arXiv:1104.5152].
- Comparisons with $SU(6) \otimes O(3)$
- Conclusions

Lattice parameters

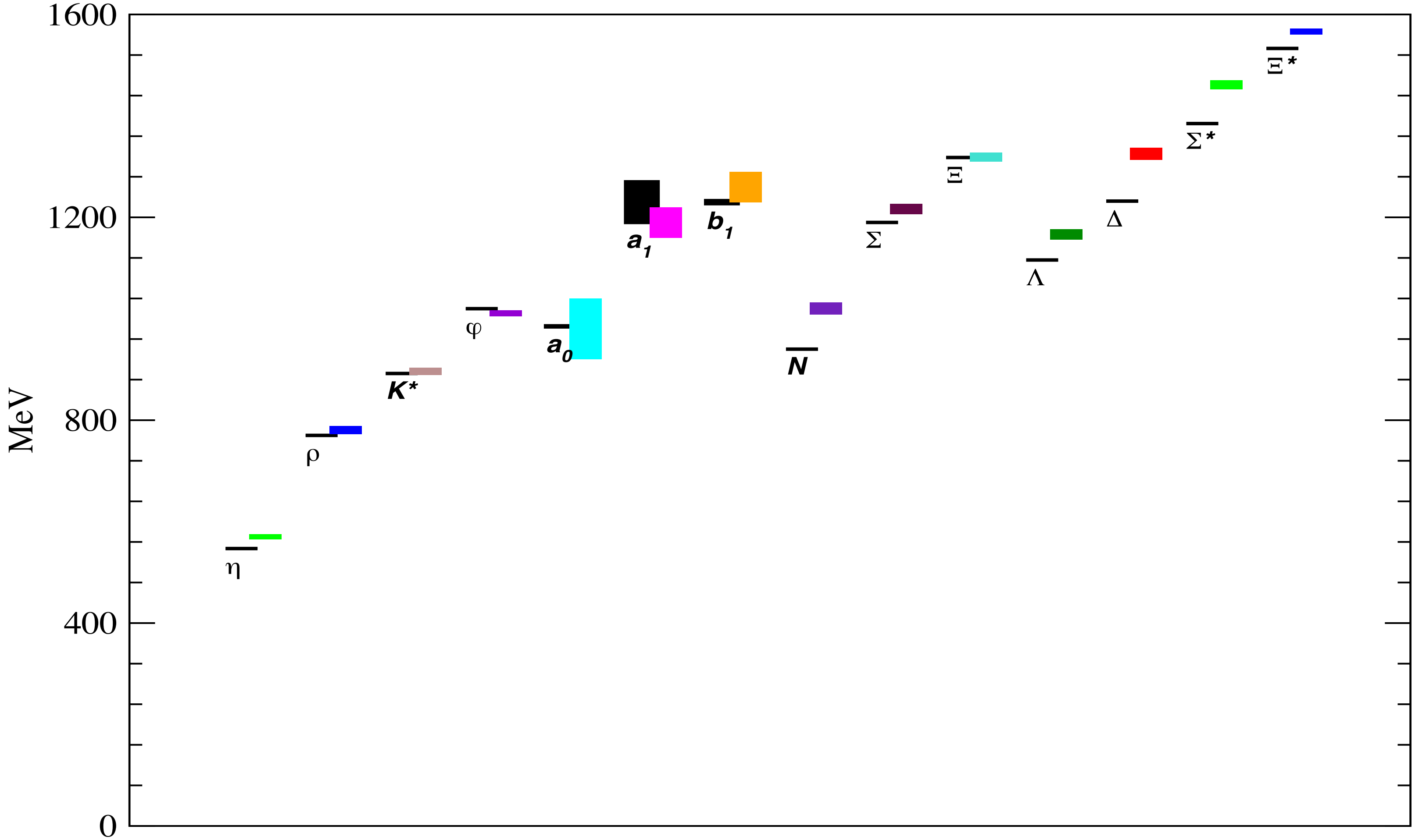
- N_f = 2+1 QCD
  - Gauge action: Symanzik-improved
  - Fermion action: Clover-improved Wilson

- Anisotropic: a_s= 0.122 fm, a_t = 0.035 fm

ensemble	1	2	3
m_ℓ	−.0840	−.0830	−.0808
m_s	−.0743	−.0743	−.0743
Volume	$16^3\times$ 128	$16^3\times$ 128	$16^3\times$ 128
Physical volume	(2 fm)^3	(2 fm)^3	(2 fm)^3
$N_{\rm cfgs}$	344	570	481
$t_{\rm sources}$	8	5	7
m_π	0.0691(6)	0.0797(6)	0.0996(6)
m_K	0.0970(5)	0.1032(5)	0.1149(6)
m_Ω	0.2951(22)	0.3040(8)	0.3200(7)
m_π (MeV)	396	444	524

HADRON SPECTRUM COLLABORATION

H.-W. Lin *et al.* Phys. Rev. D79, 034502 (2009).



Tuning of m_ℓ and m_s yields a good account of hadron masses

Limitations

- Three-quark operators:
  - No multiparticle operators
  - No clear evidence for multiparticle states: πN, etc.
- One (small) volume and one total momentum $P = 0$: No extrapolations or δ's
- $m_\pi = 396$, 444, 524 MeV : Energies generally are high
- The three-quark states essentially are stable; decays are suppressed.

Computational Resources

- USQCD allocations
- Jefferson Laboratory GPUs and HPC clusters
- and the Chroma software system (Edwards *et al.*)

Standard recipe for lattice spectra

- Use interpolating field operators $B_j^\dagger(\mathbf{x}, t)$ to create three-quark baryons.
- Construct operators so that they transform as irreps of cubic group
- Make smooth operators i.e., smear them over many lattice sites
  - Project operators to low eigenmodes of covariant lattice Laplacian
  - Peardon, *et al.*, Phys. Rev. D80, 054506 (2009)
- Matrices of correlation functions: $C_{ij}(t) = \sum_x < 0|B_i(\mathbf{x}, t) B_j^\dagger(\mathbf{0}, 0)|0 >$
  - $C_{ij}(t) \sim < i|e^{-Ht}|j >$
- Diagonalize matrices to get principal eigenvalues: $\lambda_{\mathfrak{n}}(t, t_0)$
  - Principal eigenvalues separate the decays of N eigenstates: $e^{-m_{\mathfrak{n}}(t-t_0)}$
- Fit them & extract masses, $m_{\mathfrak{n}}$.

Contamination from states outside the diagonalization space

Expect $\lambda_{\mathfrak{n}}(t) = e^{-m_{\mathfrak{n}}(t-t_0)} + \sum_{k>N} B_k e^{-m_k(t-t_0)} + \cdots$

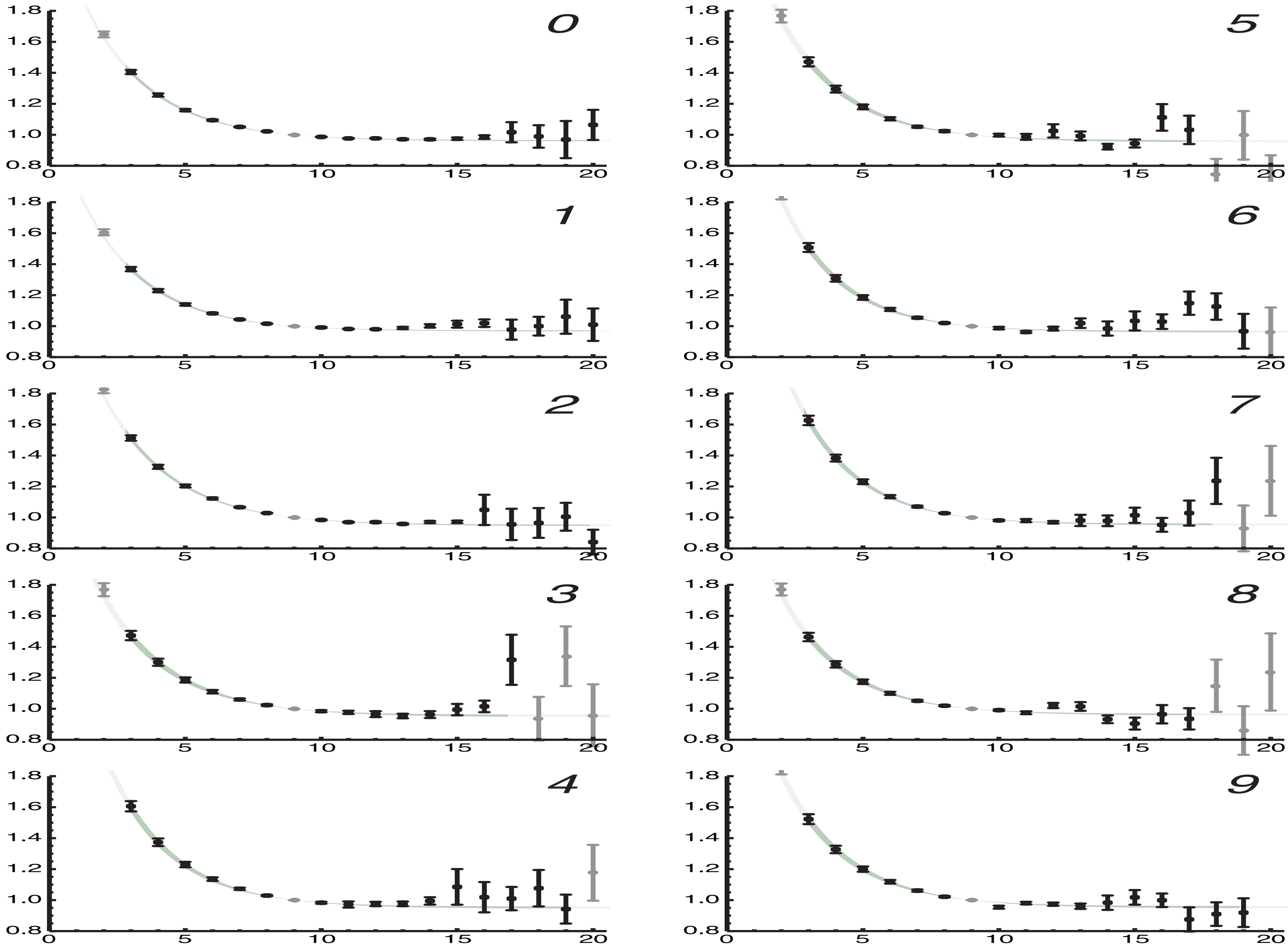
Two-exponential fits to principal eigenvalues

$$\lambda_{fit}(t) = (1 - A_{\mathfrak{n}})e^{-\mathrm{m}_{\mathfrak{n}}(t-t_0)} + A'_{\mathfrak{n}} e^{-m'_{\mathfrak{n}}(t-t_0)}$$

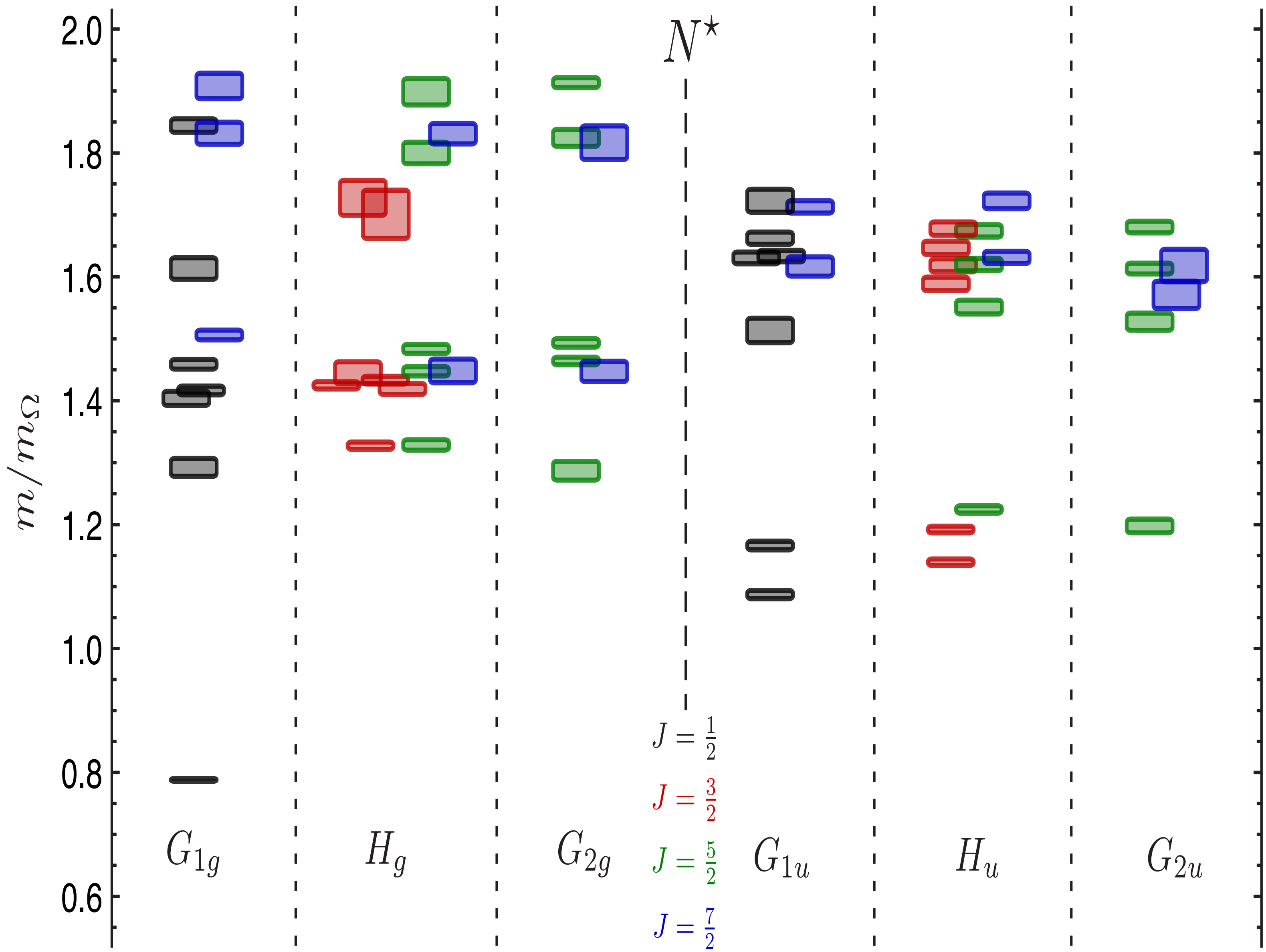
Ratio plots to show the goodness of fits

$$\frac{\lambda_{fit}(t)}{e^{-\mathrm{m}_{\mathrm{n}}(t-t_0)}}$$

Ratio tends to constant at large t



Contaminations are fit well by the 2nd exponential



N* spectrum in irreps of cubic group: $m_\pi = 396$ MeV

Results of standard recipe

- Lots of states and lots of degeneracies
- Spins are ambiguous
  - Degenerate states in $G_1,\ H,\ G_2$ irreps imply a $J = \frac{7}{2}$ state
  - or accidentally degenerate $J = \frac{1}{2}$ and $J = \frac{5}{2}$ states
- Spin identification fails because:
  - there are too many degenerate states to identify the subductions of high spins
  - lattice energies don't provide sufficient information

New recipe to identify spins

- Use operators with known spins in continuum limit
  - Incorporate covariant derivatives to realize orbital angular momenta
- Subduce the operators to irreps of cubic group
- Use spectral representation of matrices: $C_{ij}(t) = \sum_{\mathfrak{n}} Z_i^{\mathfrak{n}*} Z_i^{\mathfrak{n}} e^{-m_{\mathfrak{n}} t}$
- $Z_i^{\mathfrak{n}} =< \mathfrak{n}|B_i^{\dagger}(\mathbf{0}, 0)|0 >$ is the overlap of operator i with state $\mathfrak{n}$
- Use $Z_i^{\mathfrak{n}}$ to identify spin: spin of state $\mathfrak{n}$ is J when largest Z's are for operators subduced from spin J
  - The different lattice irreps give approximately the same overlaps
  - $E_{\mathfrak{n}}$ is the energy of a state of good J.

Construction of operators with good J in continuum

- Mesons: Dudek, *et al.*, Phys.Rev.D80:054506,2009
- Baryons: Color singlet structure for 3 quarks, symmetric in space & spin
- $\mathbf{J} = \mathbf{L} + \mathbf{S}$ with
  - S = $\frac{1}{2}$ or $\frac{3}{2}$ from quark spins
  - L = 1 or 2 from covariant derivatives
  - J = $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$ and $\frac{7}{2}$
  - Upper ($\rho = +$) and lower ($\rho = -$) components of Dirac spinors
- Lots of operators $\mathcal{O}^{[J,M]}$ with good spin in continuum limit
- Feynman, Kislinger and Ravndal formalism for quark states applied to operator construction, except $SU(12) \otimes O(3)$

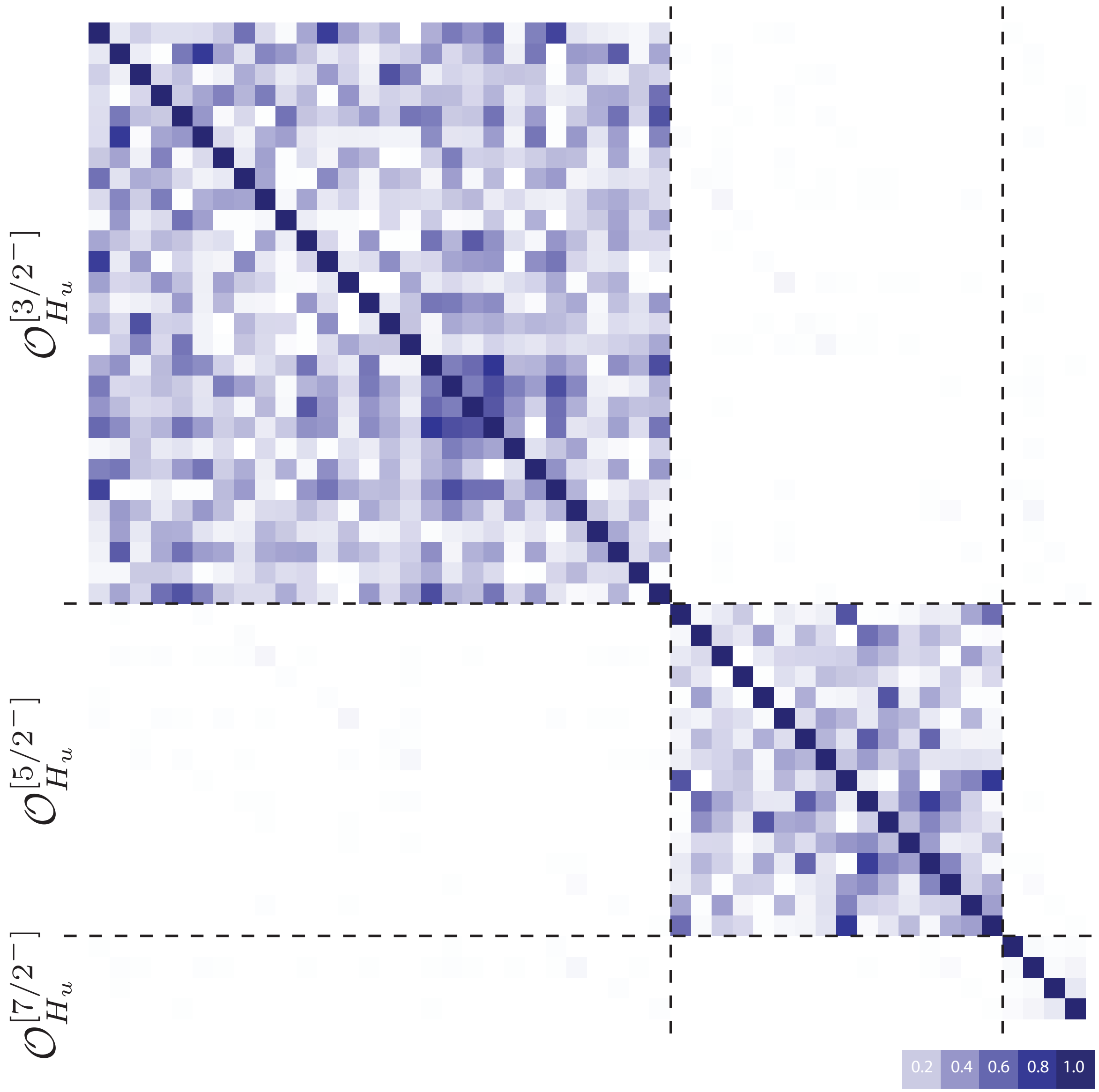
Subduction to irreps of cubic group

- Cubic group irreps Λ and rows r provide orthogonal basis on lattice
- In quantum mechanics, subduction is a change of basis $|J, M\rangle \rightarrow |\Lambda, r; J\rangle$.
- $|\Lambda, r; J\rangle = \sum_M |J, M\rangle\langle J, M|\Lambda, r; J\rangle$

  $= \sum_M |J, M\rangle \; S^{J,M}_{\Lambda,r}$.
- Subduction matrices: $S^{J,M}_{\Lambda,r}$
- Subduced operators: $\mathcal{O}^{[\Lambda,r;J]} = \sum_M \mathcal{O}^{[J,M]} S^{J,M}_{\Lambda,r}$
- When rotational symmetry is broken weakly,

  $\langle 0|\mathcal{O}^{[\Lambda,r;J]}(t)\mathcal{O}^{[\Lambda,r;J']\dagger}(0)|0 > \approx \delta_{J,J'}$ is block diagonal in J.

Matrix C_{ij} is block diagonal approximately



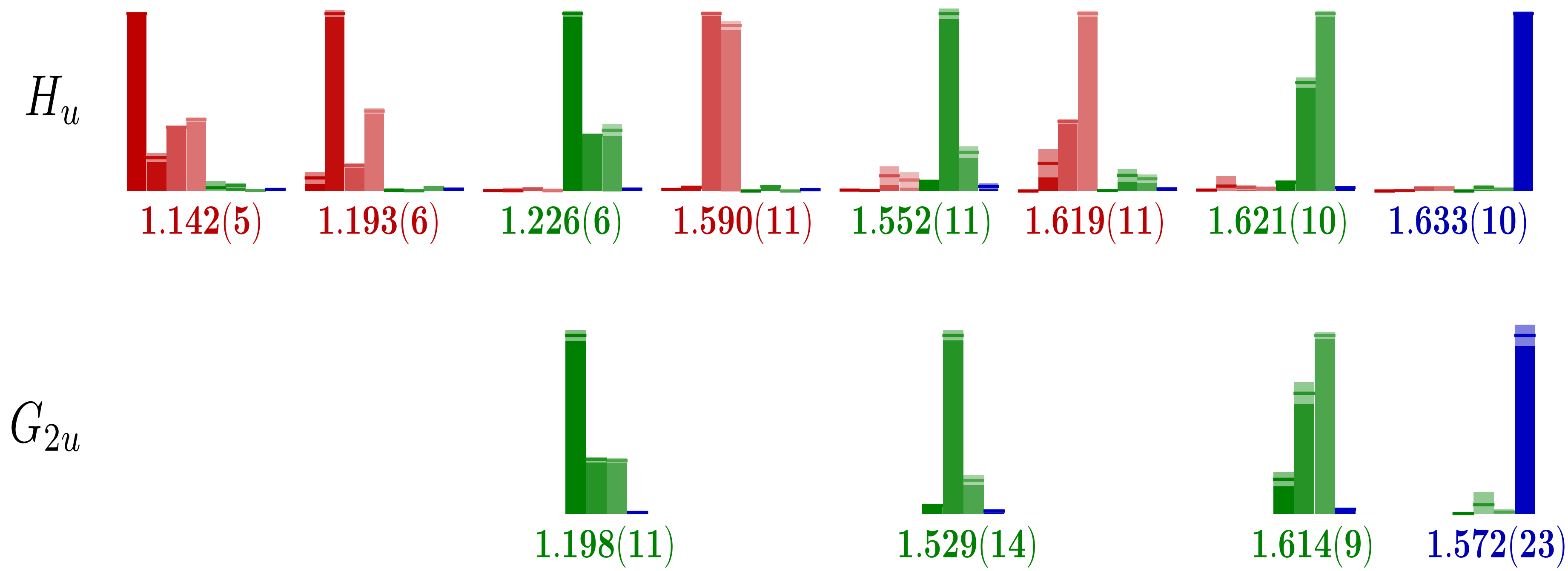
Magnitude of matrix elements in a matrix of correlation functions at timeslice 5.

Reasons for approximate rotational invariance

- Rotational symmetry is broken at $\mathcal{O}(\mathbf{a}^2)$ by lattice action
- Lattice spacing is 0.12 fm
- Typical hadron size is 1 fm
- Smearing makes operators smooth on the hadron size scale
- Estimate: $\mathcal{O}(a^2) \approx \left(\frac{0.12fm}{1.0fm}\right)^2 \approx$ **0.015**
- For hadrons, rotational symmetry is broken weakly.

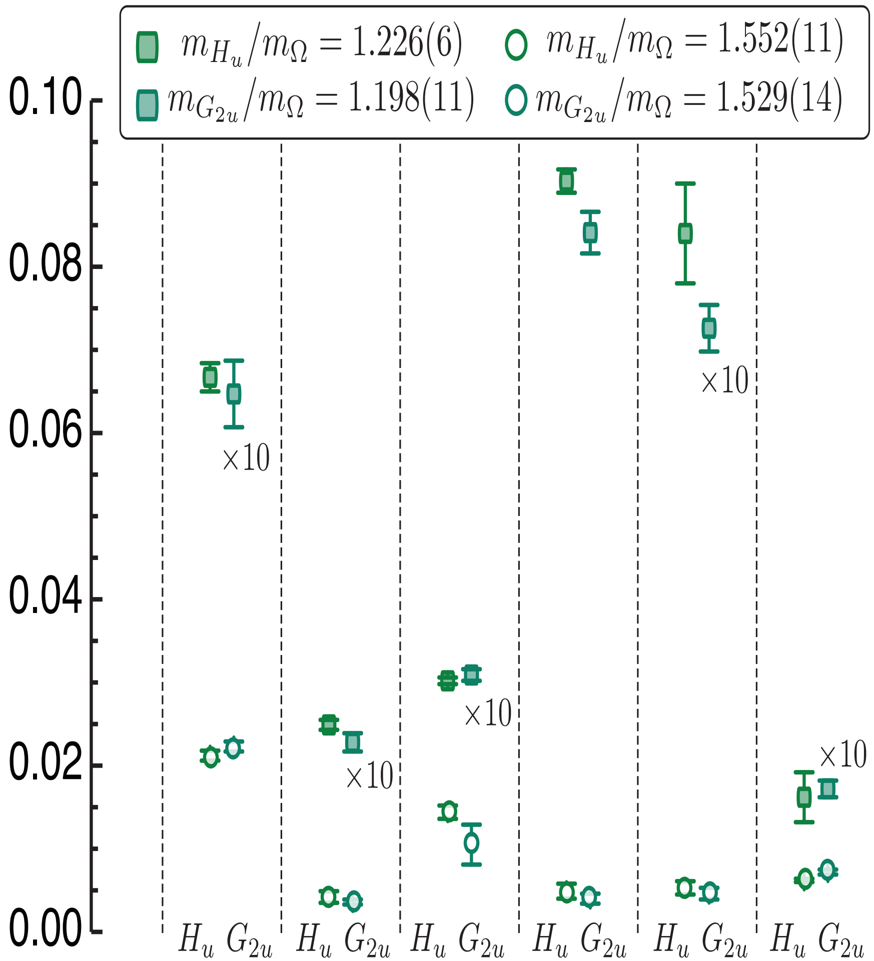
Spin identification: $Z_i^{\mathfrak{n}}$ values show which operators dominate each state

- $\left(N_{\mathsf{M}} \otimes \left(\tfrac{1}{2}^{+}\right)^{1}_{\mathsf{M}} \otimes D^{[1]}_{L=1,\mathsf{M}}\right)^{J=\frac{3}{2}}$
- $\left(N_{\mathsf{M}} \otimes \left(\tfrac{3}{2}^{+}\right)^{1}_{\mathsf{S}} \otimes D^{[1]}_{L=1,\mathsf{M}}\right)^{J=\frac{3}{2}}$
- $\left(N_{\mathsf{M}} \otimes \left(\tfrac{1}{2}^{-}\right)^{1}_{\mathsf{M}} \otimes D^{[2]}_{L=2,\mathsf{S}}\right)^{J=\frac{3}{2}}$
- $\left(N_{\mathsf{M}} \otimes \left(\tfrac{3}{2}^{-}\right)^{1}_{\mathsf{M}} \otimes D^{[2]}_{L=2,\mathsf{S}}\right)^{J=\frac{3}{2}}$
- $\left(N_{\mathsf{M}} \otimes \left(\tfrac{3}{2}^{+}\right)^{1}_{\mathsf{S}} \otimes D^{[1]}_{L=1,\mathsf{M}}\right)^{J=\frac{5}{2}}$
- $\left(N_{\mathsf{M}} \otimes \left(\tfrac{1}{2}^{-}\right)^{1}_{\mathsf{M}} \otimes D^{[2]}_{L=2,\mathsf{S}}\right)^{J=\frac{5}{2}}$
- $\left(N_{\mathsf{M}} \otimes \left(\tfrac{3}{2}^{-}\right)^{1}_{\mathsf{M}} \otimes D^{[2]}_{L=2,\mathsf{S}}\right)^{J=\frac{5}{2}}$
- $\left(N_{\mathsf{M}} \otimes \left(\tfrac{3}{2}^{-}\right)^{1}_{\mathsf{M}} \otimes D^{[2]}_{L=2,\mathsf{S}}\right)^{J=\frac{7}{2}}$

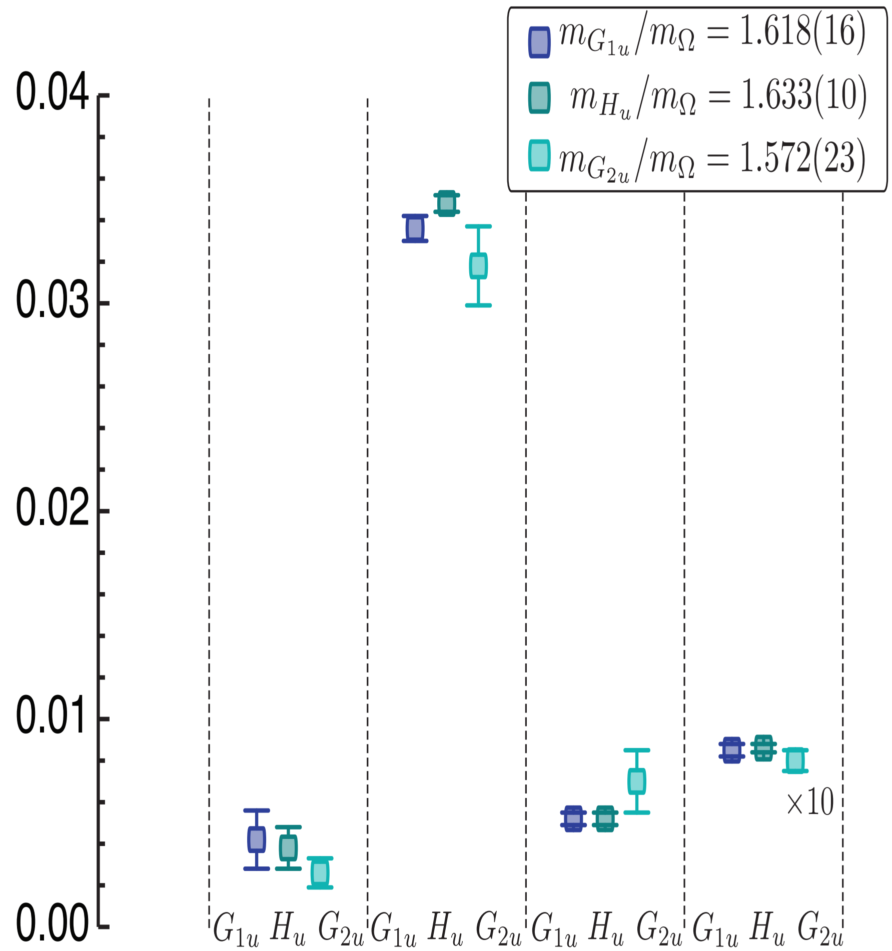


Spin identification: Nearly the same Z in each lattice irrep that belongs to the subduction of J

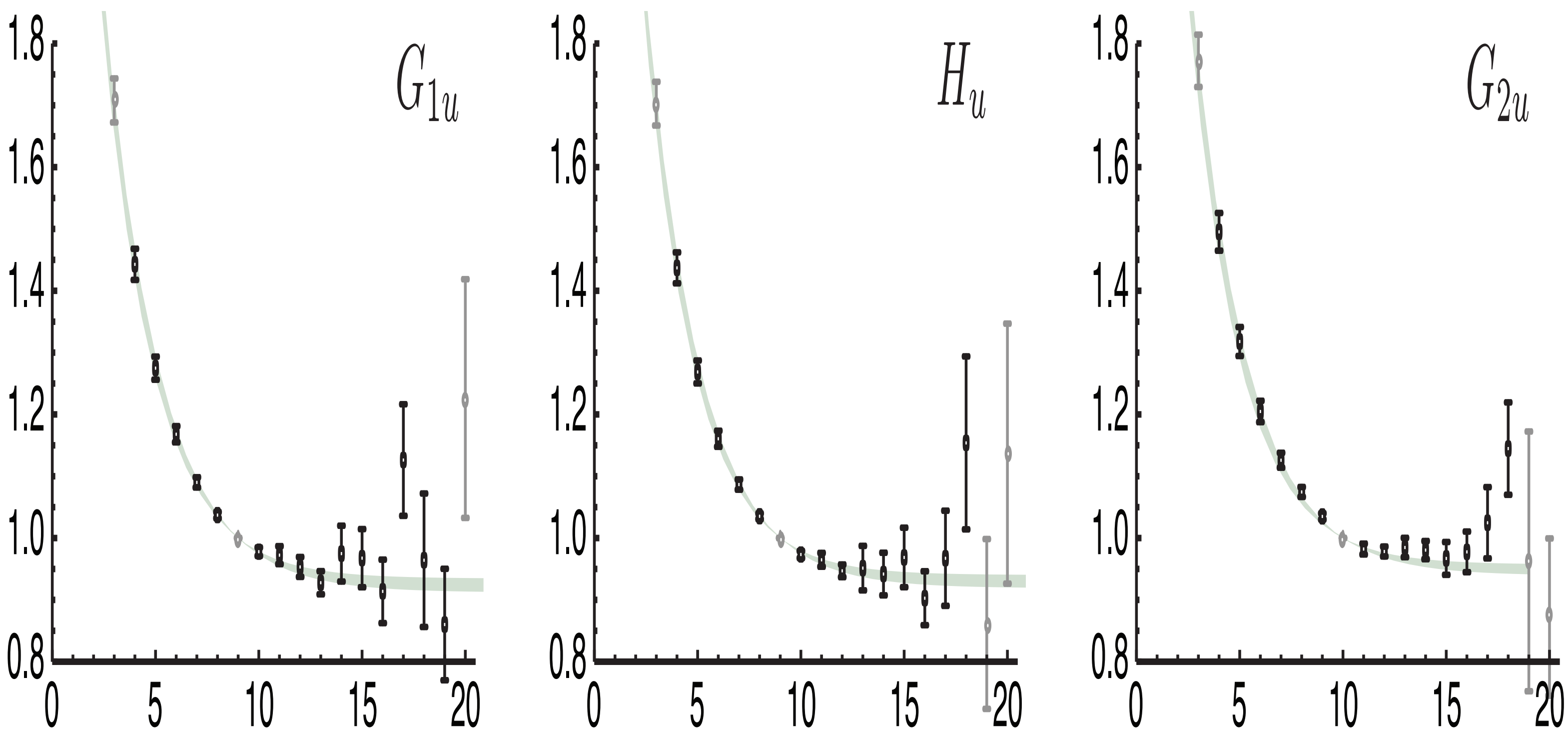
$\mathbf{J} = \frac{5}{2}$



$\mathbf{J} = \frac{7}{2}$



Joint fits of G_{1u}, H_u, G_{2u} principal correlators to a common mass determine the $\mathrm{J} = \frac{7}{2}^{-}$ energies



Spin identification of baryon excited states

- The spin of a lattice excited state is equal to J when the state is created predominantly by operators subduced from continuum spin J.

- Approximately the same Z value is obtained in each lattice irrep that belongs to the subduction of a single J value.

- Z values often are large only for a few operators, allowing interpretation of the states

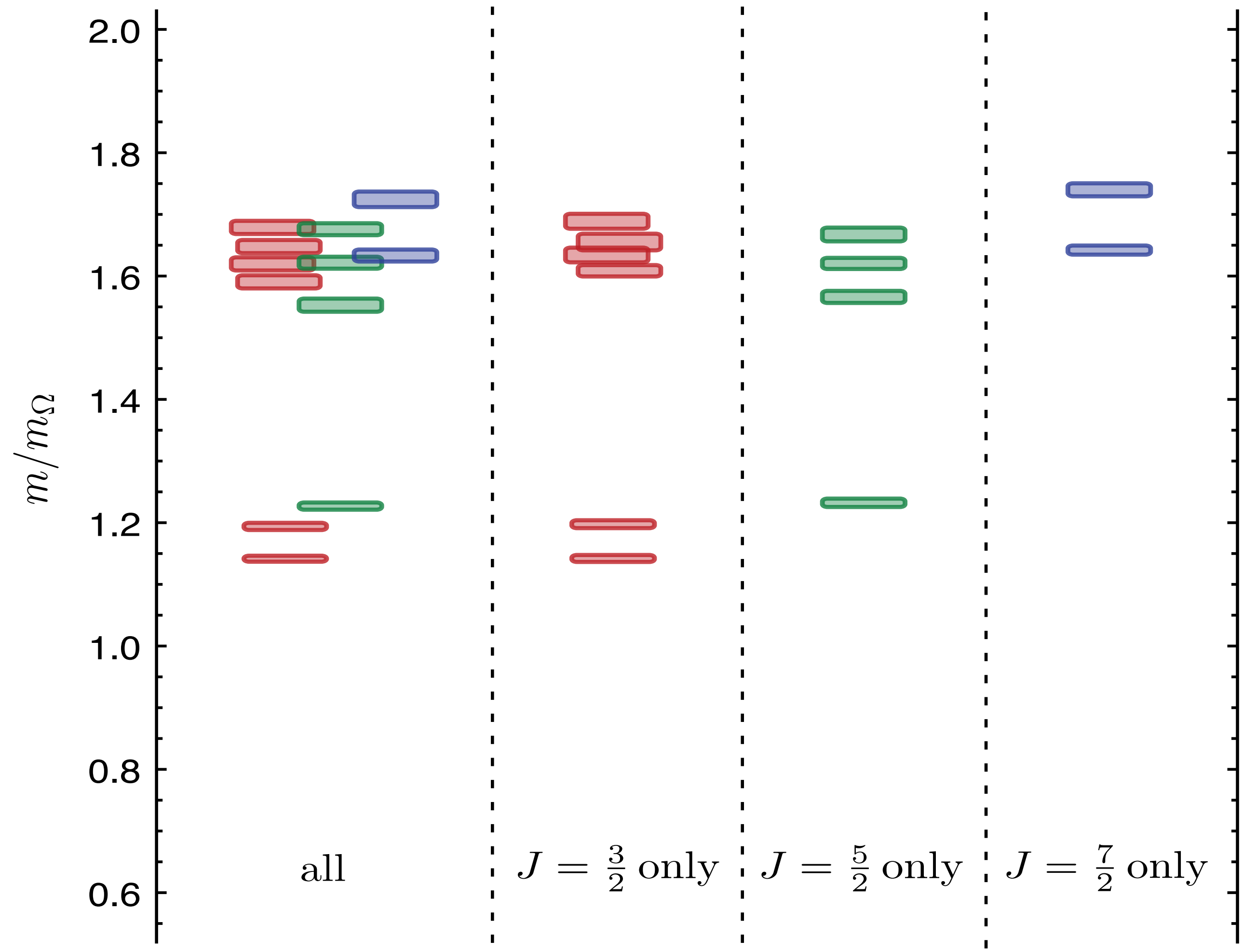
- Spin identification is reliable at the scale of hadrons

Spectral test of approximate rotational invariance

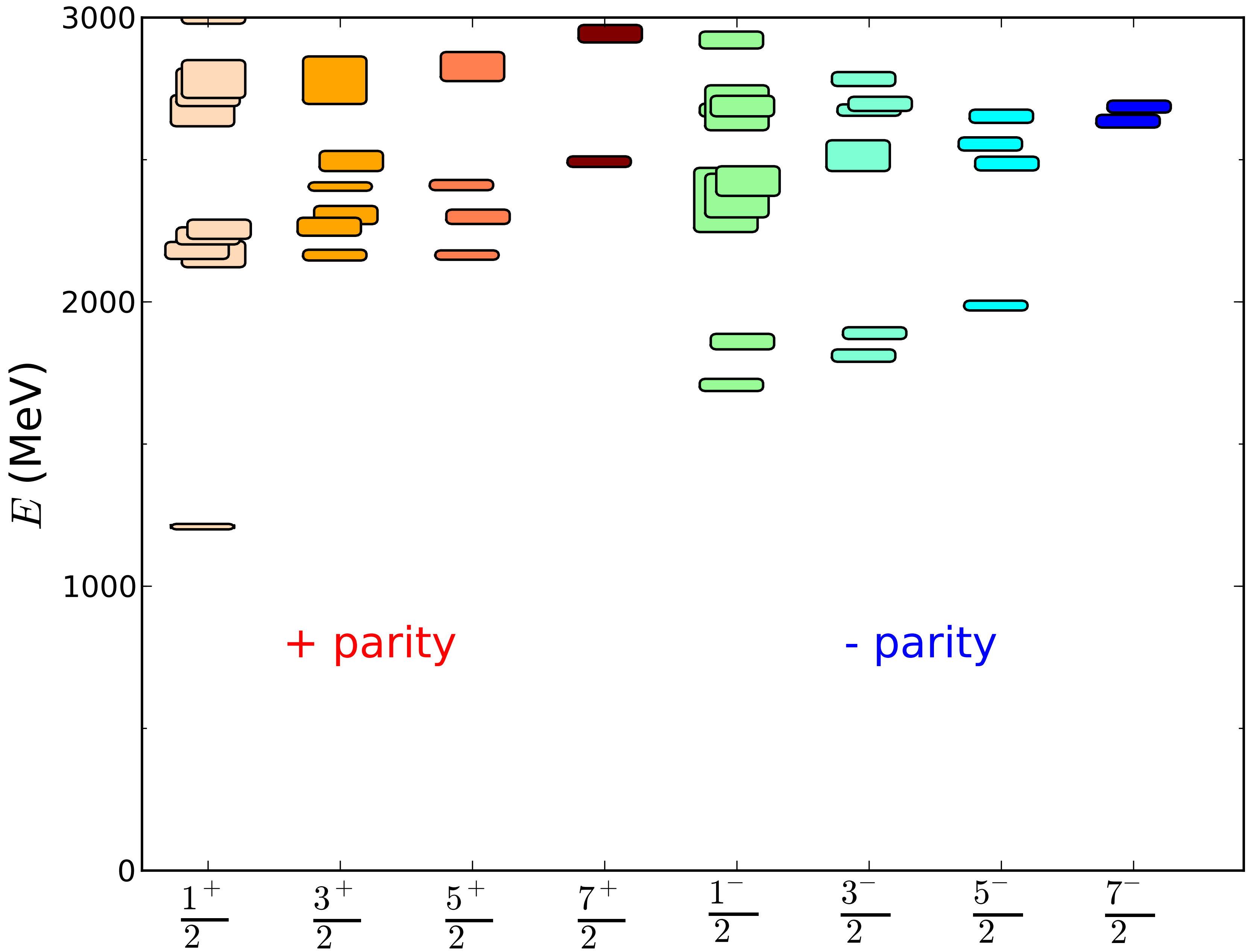
- Rotational invariance implies zero couplings between different J's, so $C \propto \delta_{J,J'}$ is block diagonal
- We find small violations of block diagonality in C.
- Does the spectrum exhibit approximate rotational invariance?
- Calculate energies including $J \neq J'$ couplings
- Calculate energies omitting $J \neq J'$ couplings

Approximate rotational invariance in spectrum,

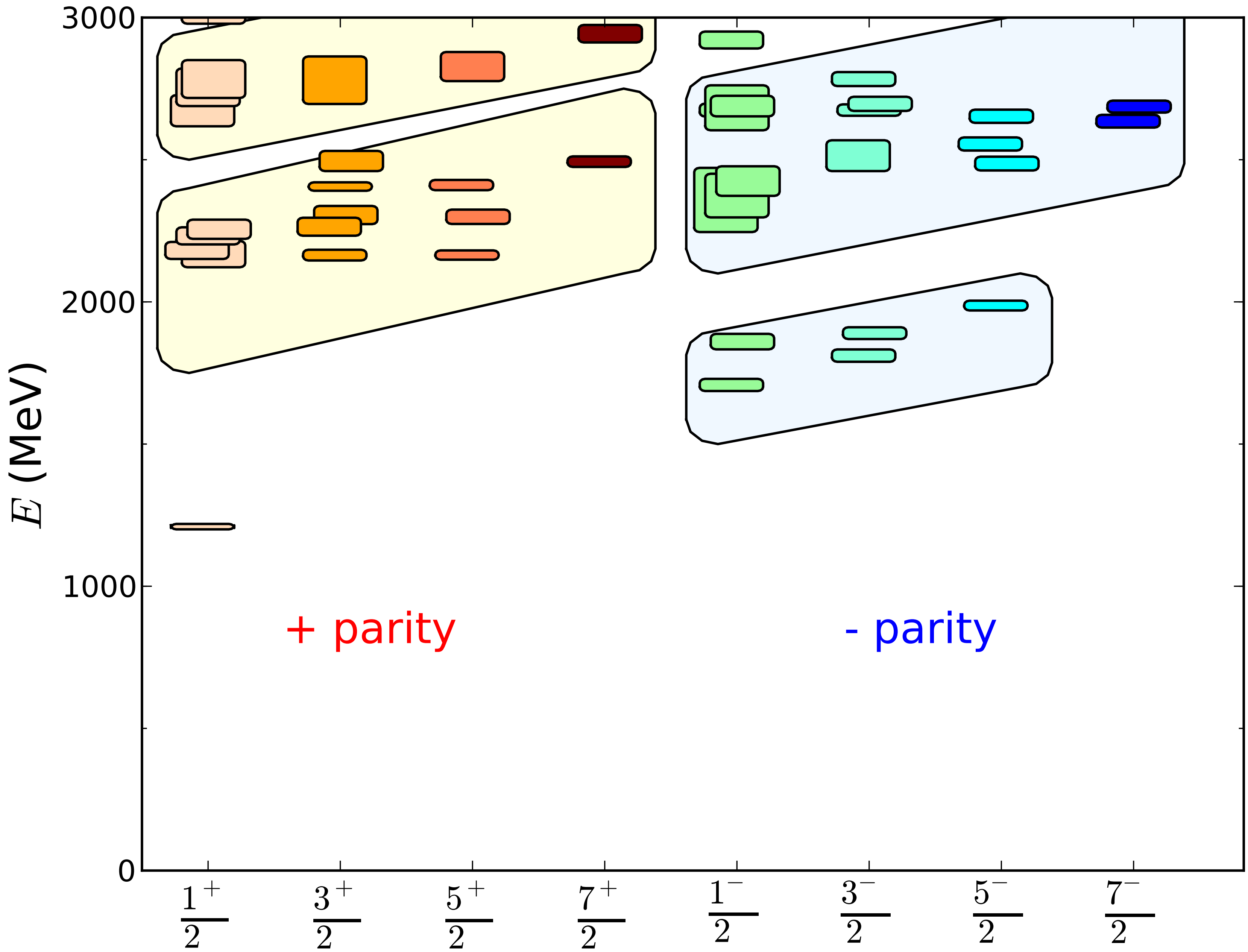
all 48 H_u 28 J=$\frac{3}{2}$ 16 J=$\frac{5}{2}$ 4 J=$\frac{7}{2}$



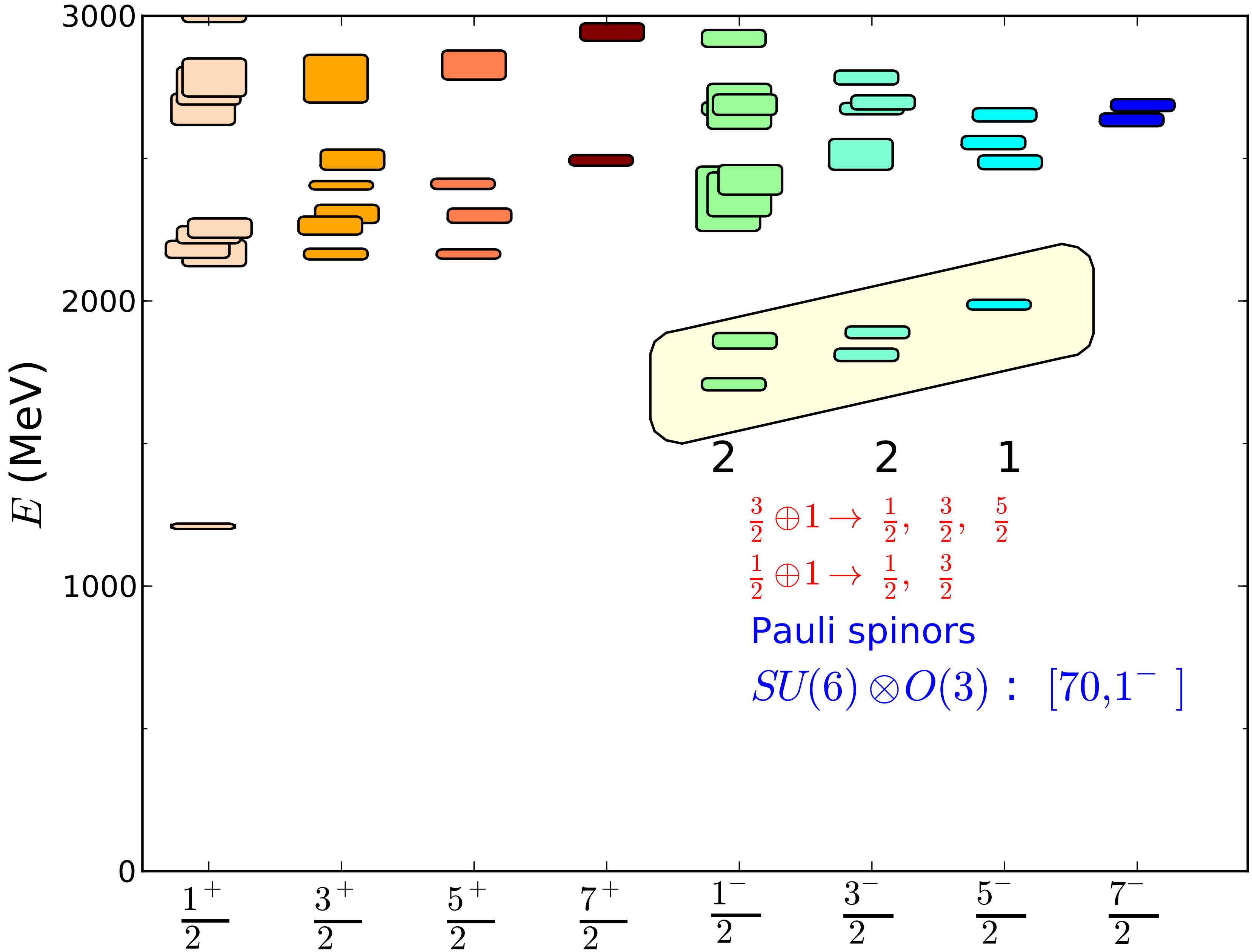
$\approx$ same energies with and without $J \neq J'$ couplings



Lattice N* excited states vs. J^P : m_π = 396 MeV

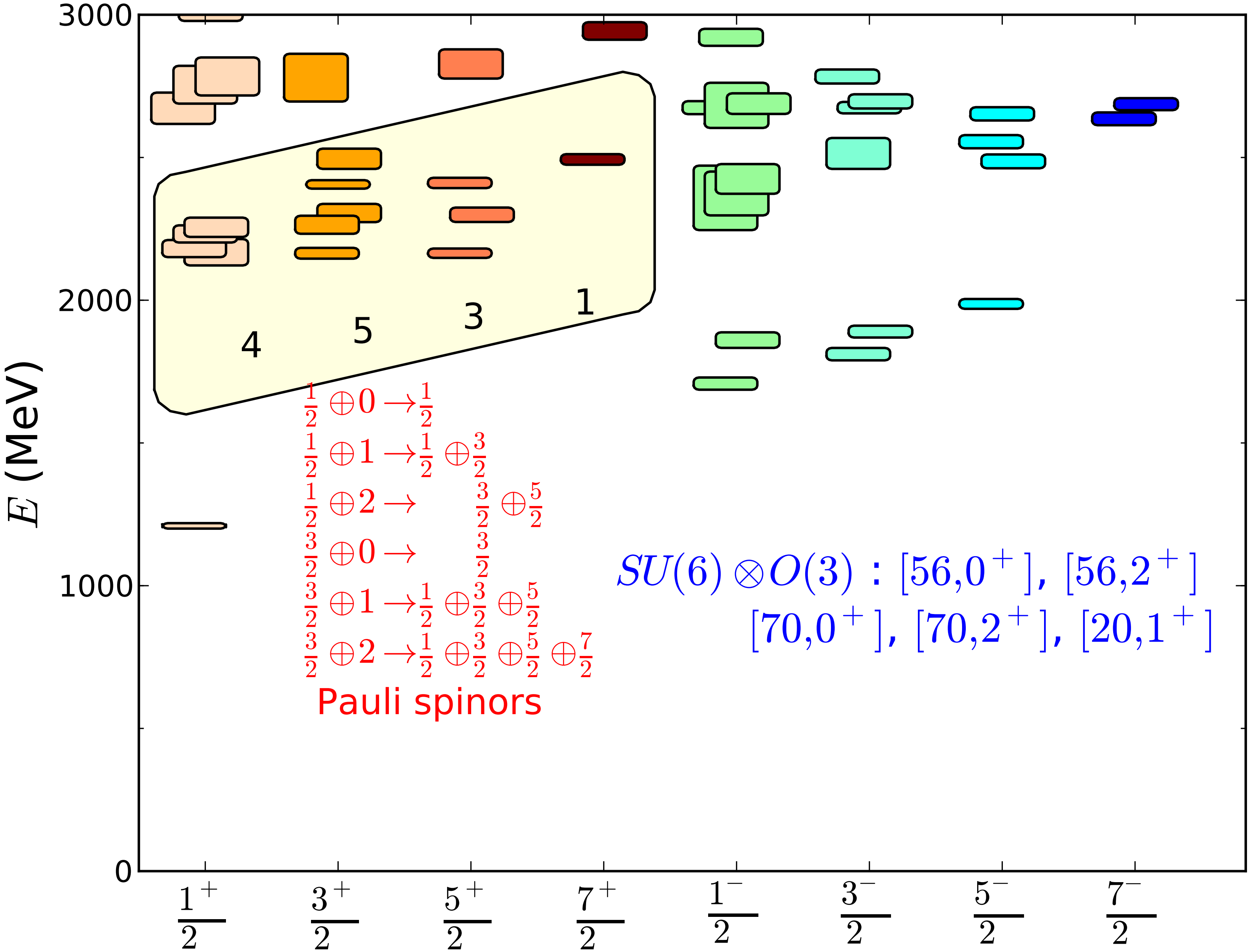


Lattice N^* spectrum: bands with $+$ and $-$ parity.



1^{st} N^{*-} band: $SU(6) \otimes O(3)$ states

see also R. Edwards, I-C, 15:45



1^{st} N^{*+} band: $SU(6)\otimes O(3)$ states

Overall pattern of N* states

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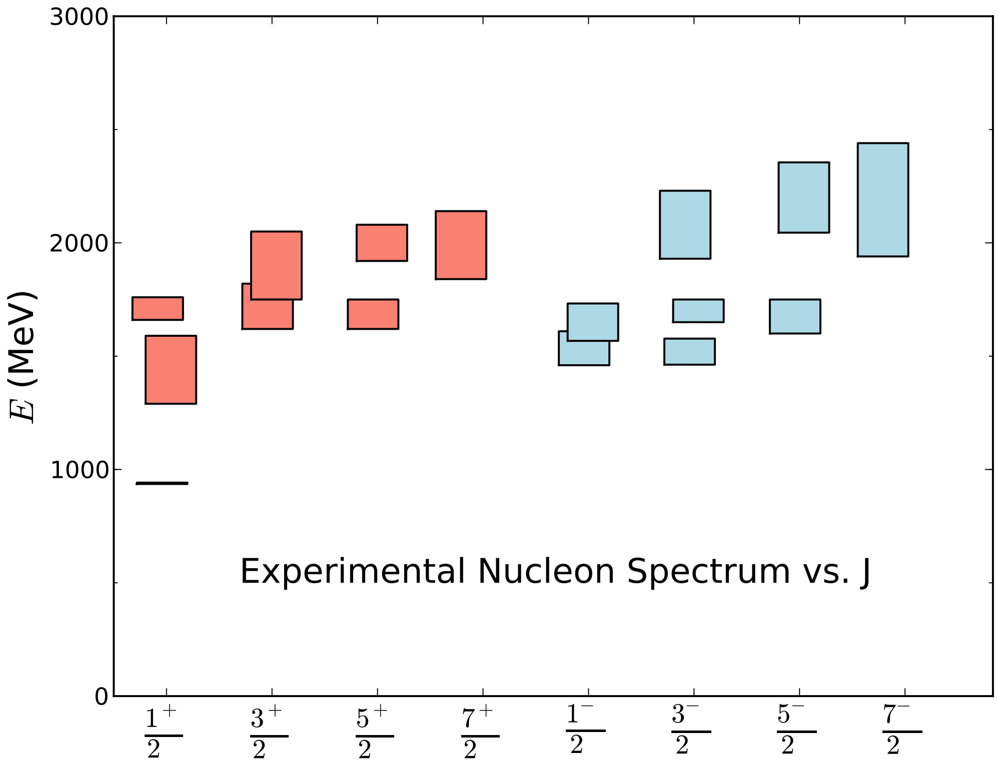
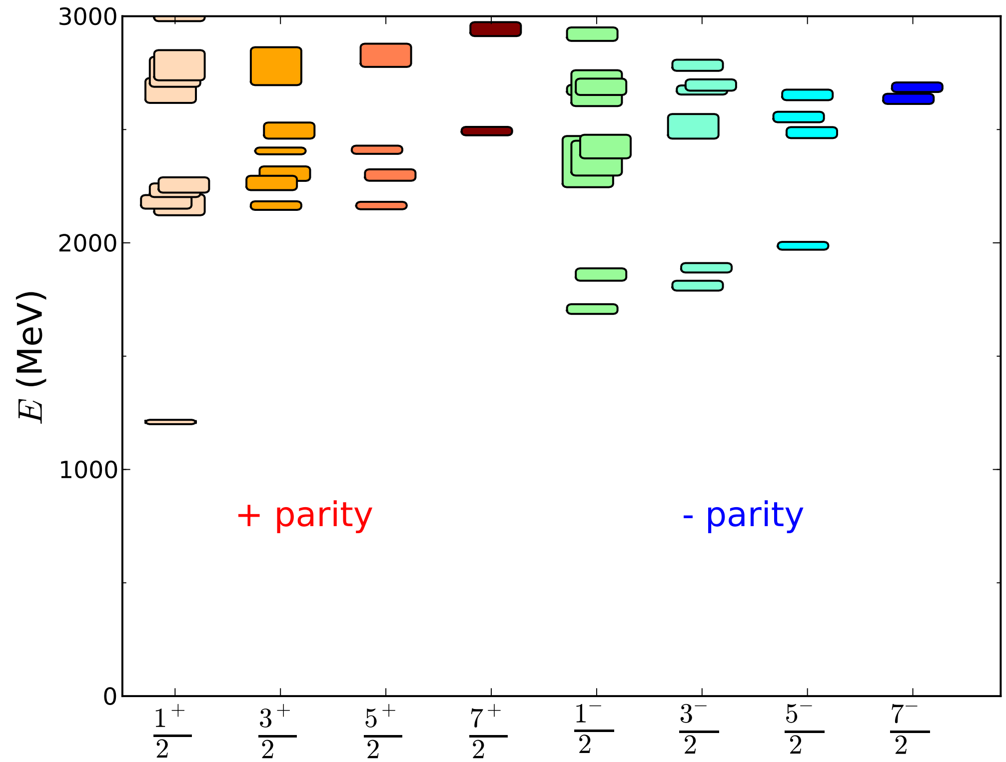
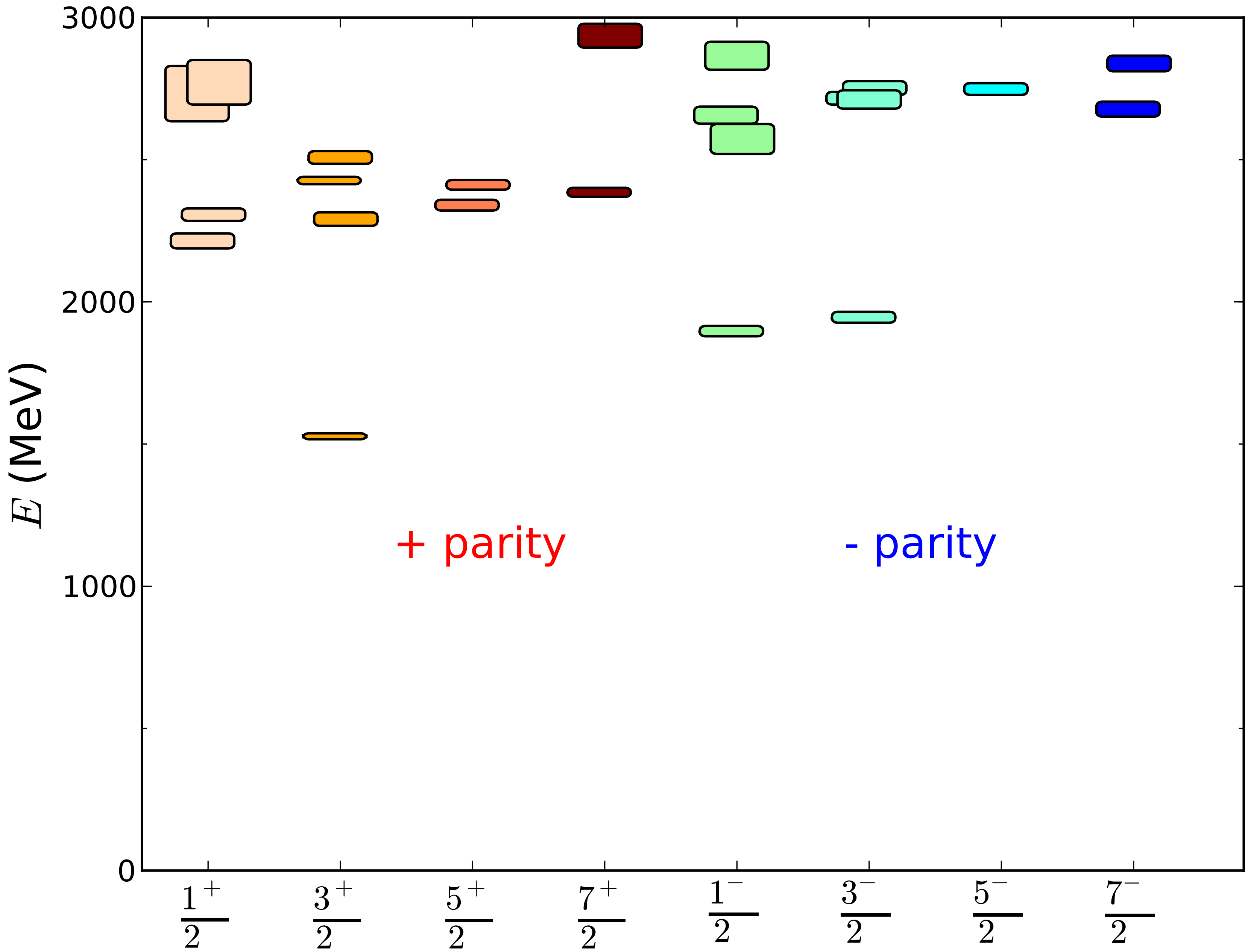


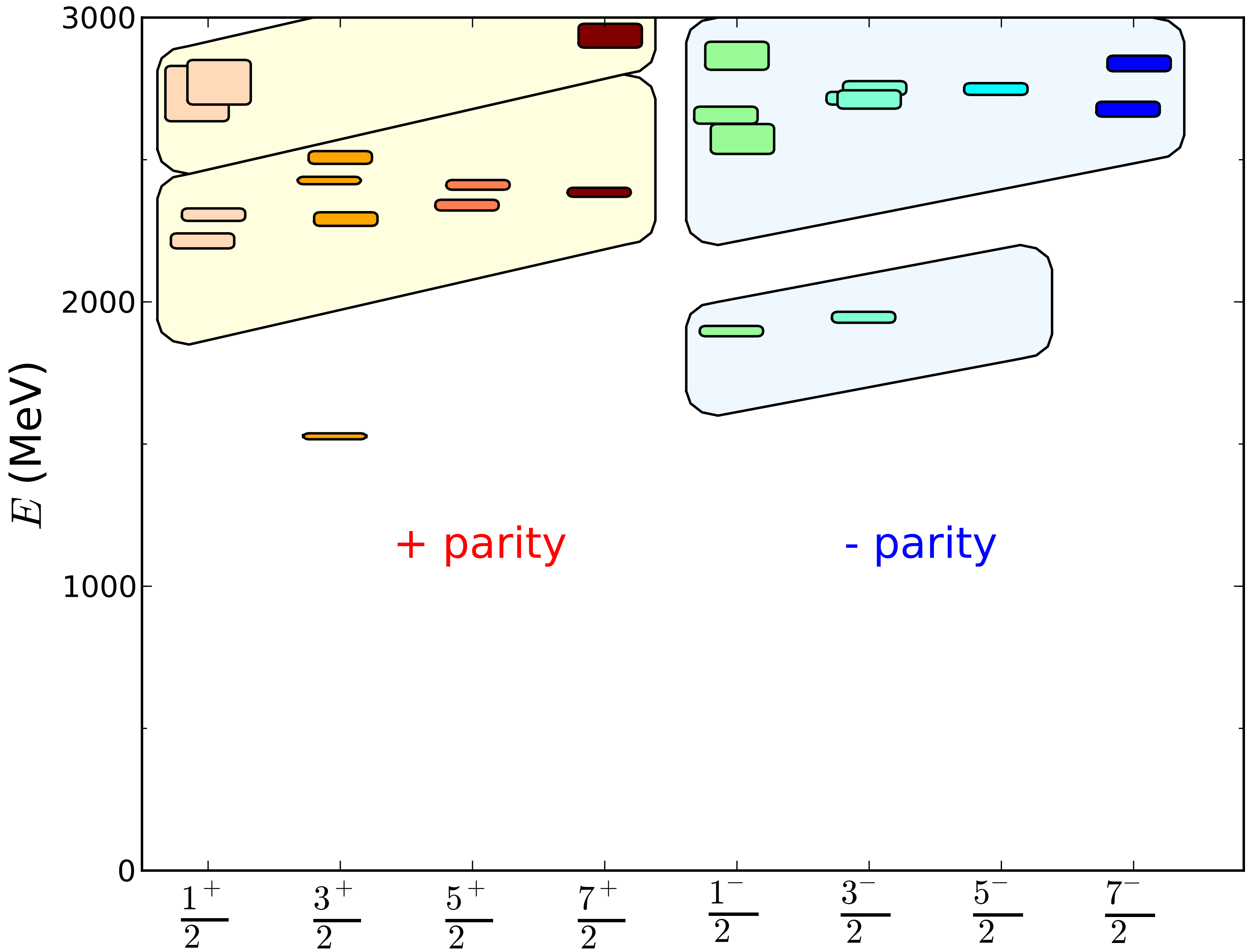
Lattice



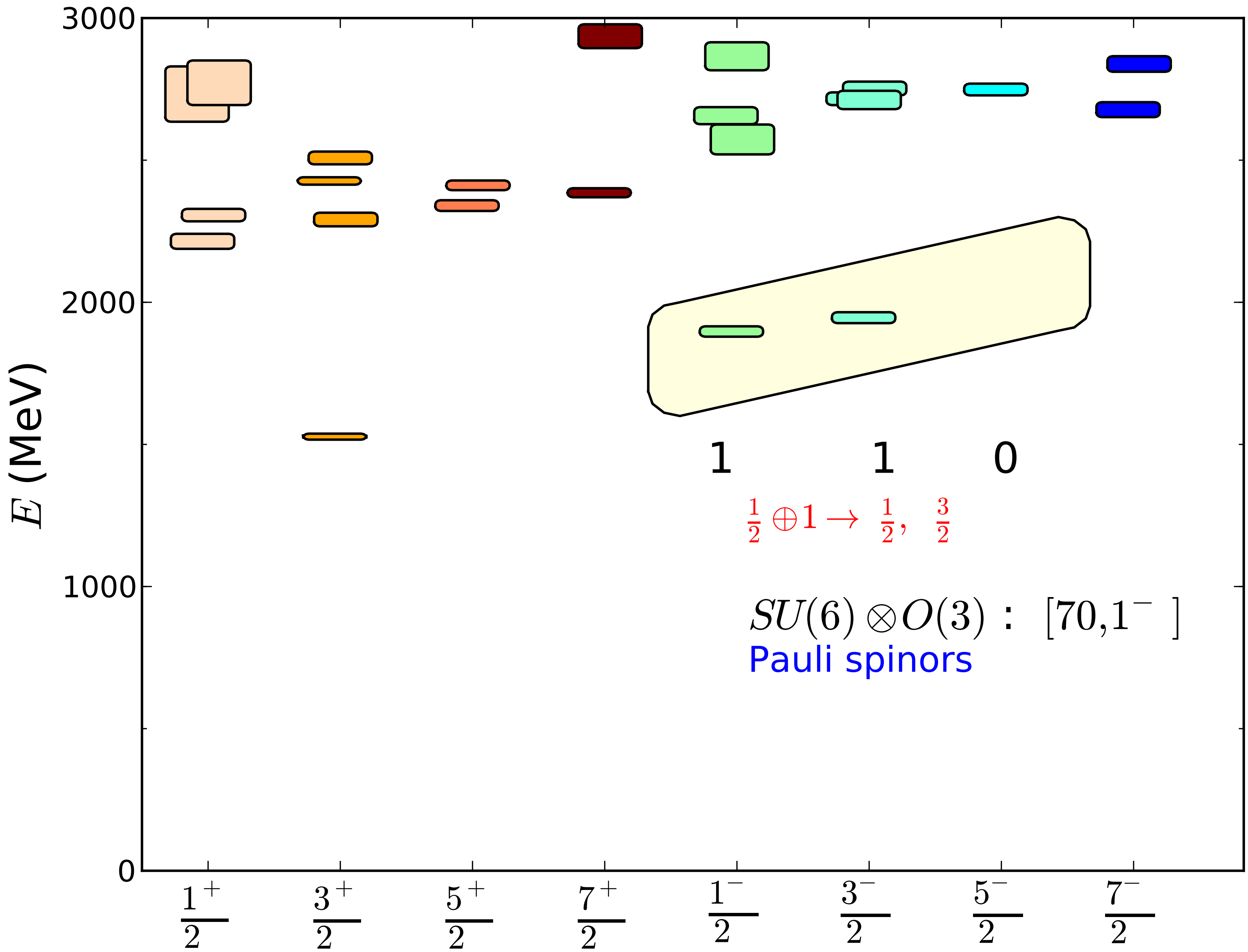
Many more states in the lattice spectrum.



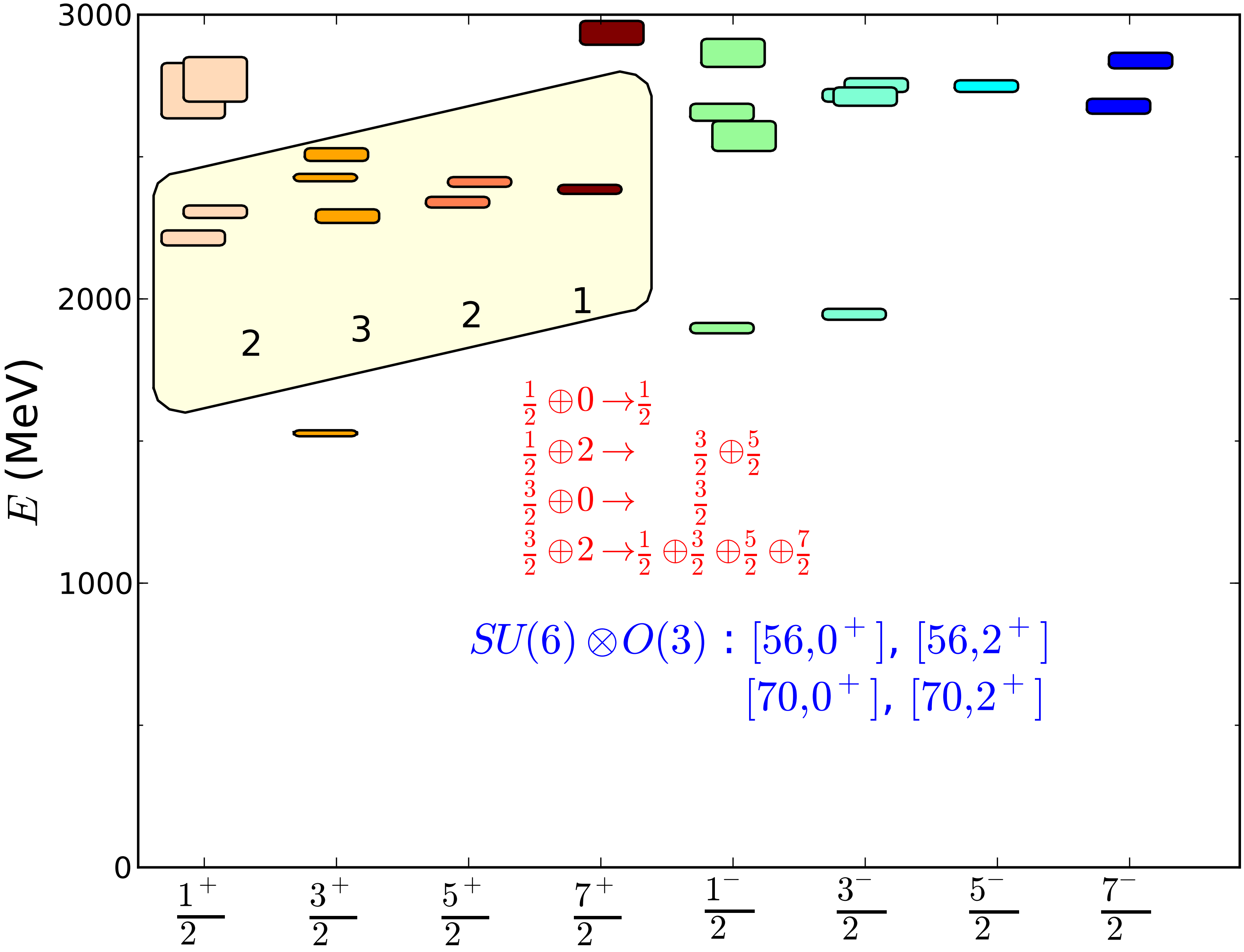
Lattice Δ excited states vs. J^P : $m_\pi = 396$ MeV



Lattice Δ spectrum : bands of $+$ and $-$ parity states



1^{st} Δ^- band : $SU(6) \otimes O(3)$ states



1^{st} Δ^+ band : $SU(6) \otimes O(3)$ states

Patterns of Δ states

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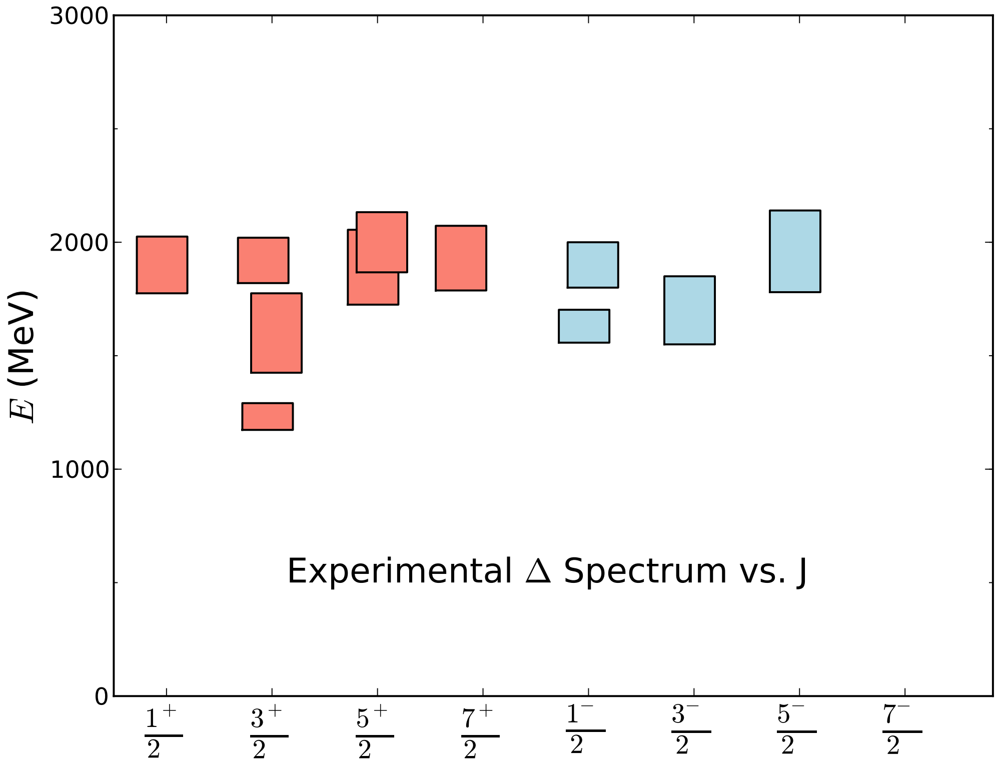
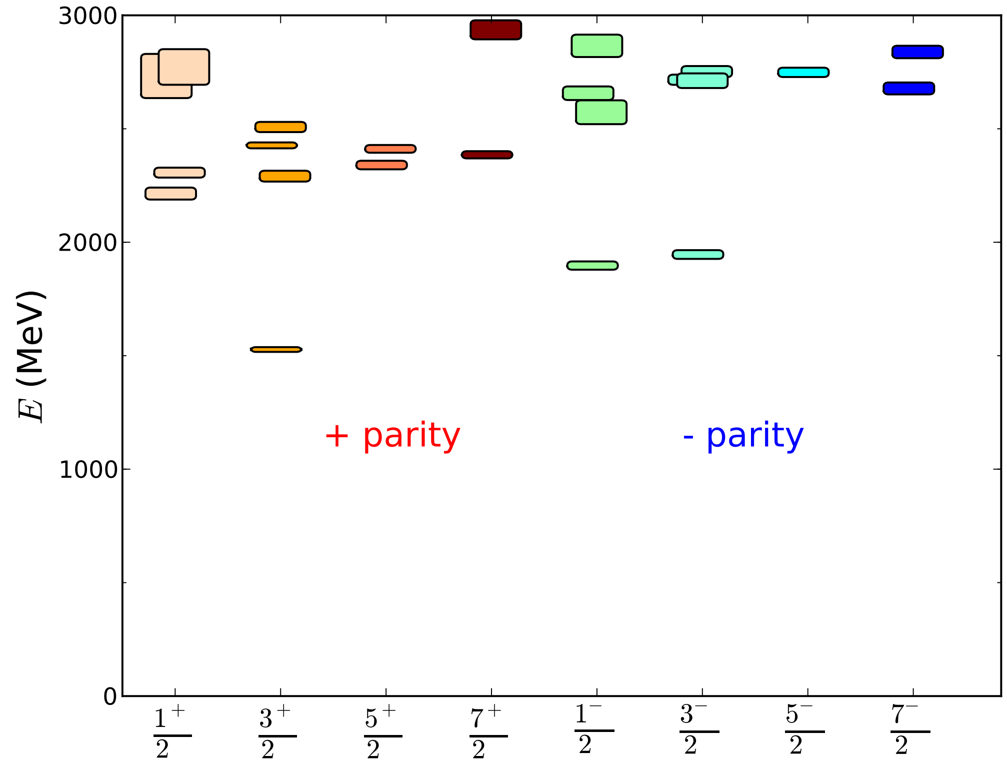


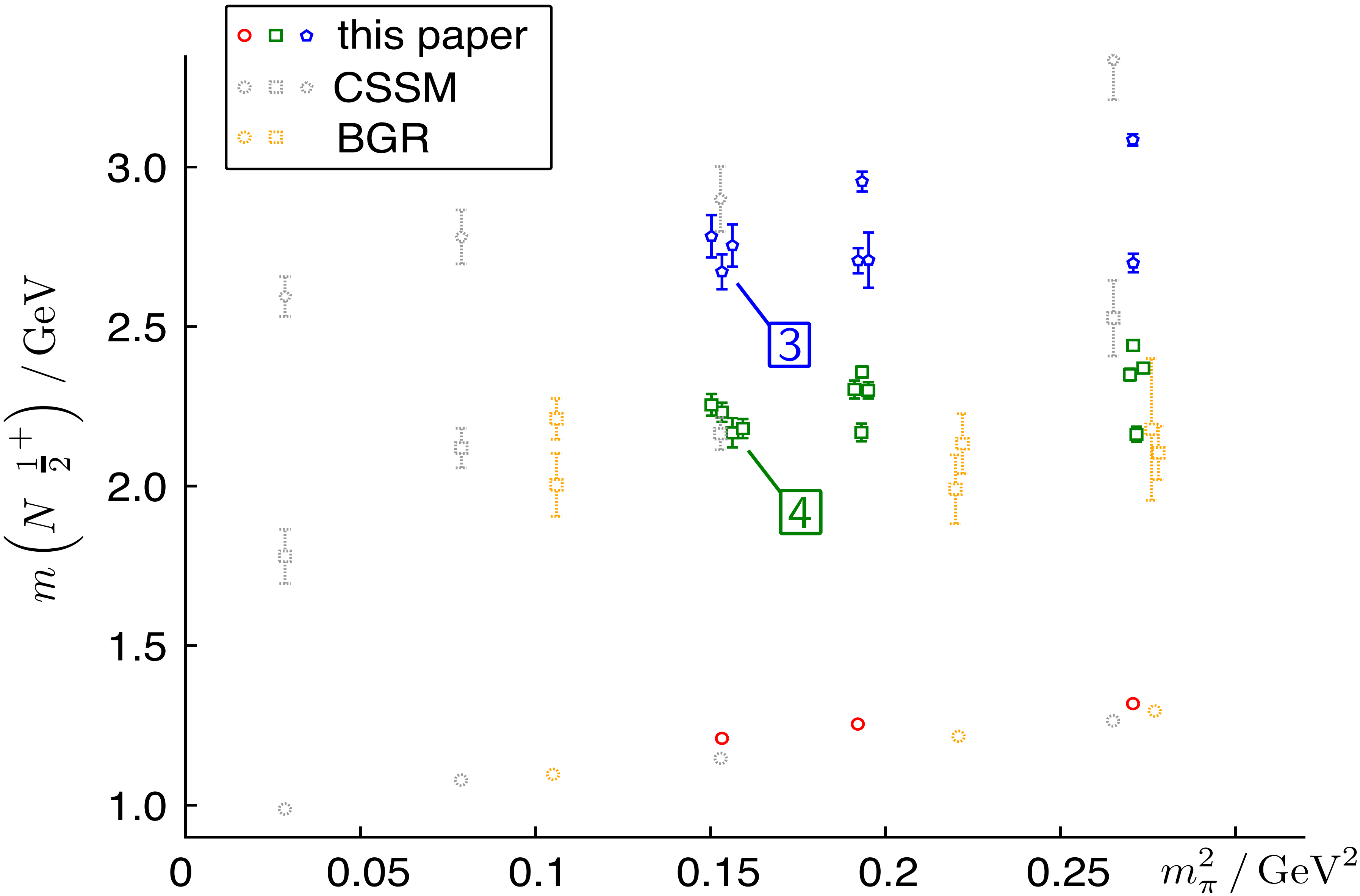
Lattice



Many more states in the lattice spectrum.

Comparison of lattice results for Roper resonance

see also D. Leinweber talk in session I-C today at 17:05



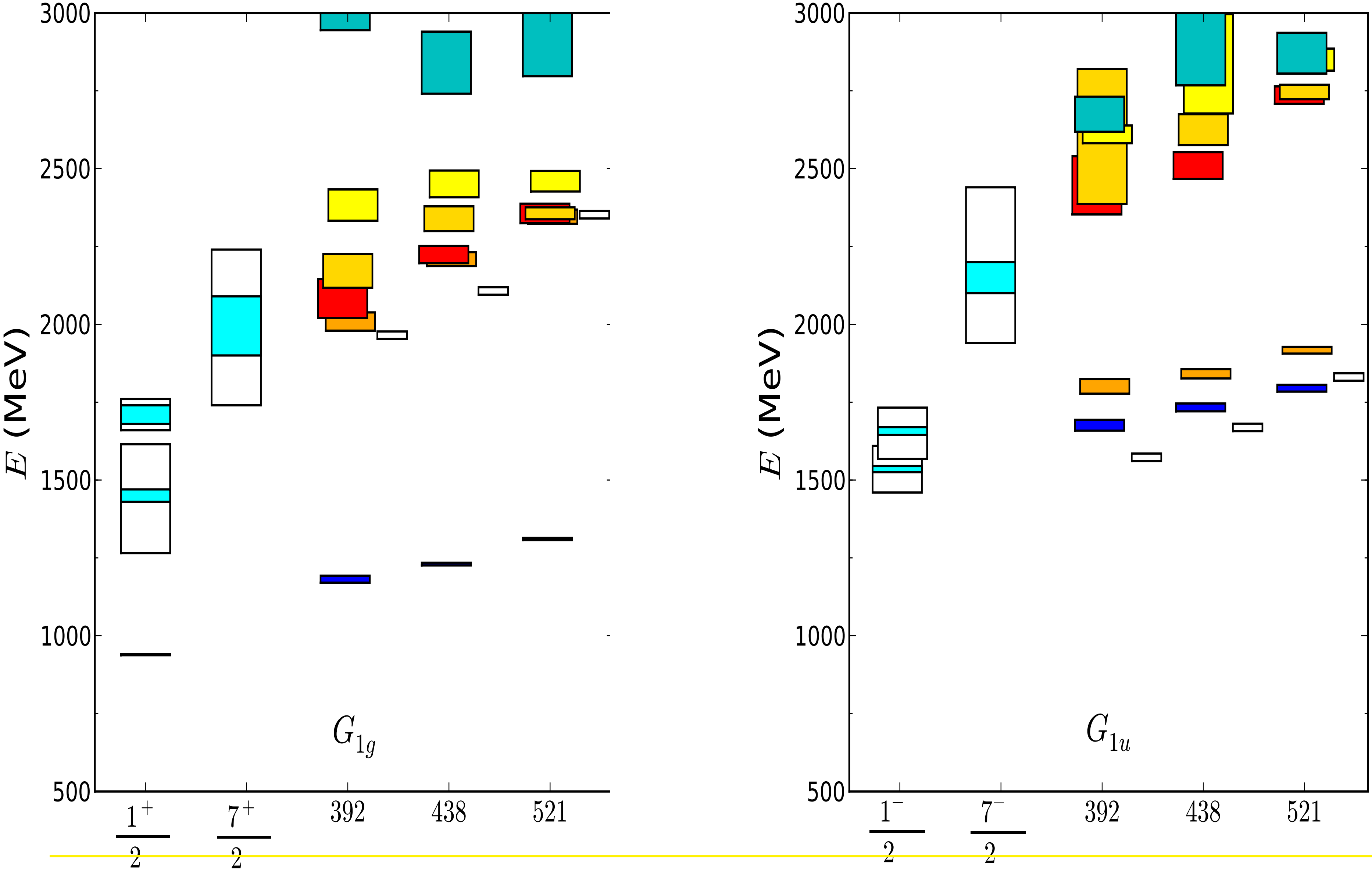
Does the Roper resonance have a complex structure?

Conclusions

- Spins are identified reliably up to $J = \frac{7}{2}$
  - Covariant derivatives provide orbital angular momenta
  - Approximate rotational invariance is realized at the scale of hadrons
  - Spectral overlaps Z identify which J values dominate a state
- Low N* and Δ bands: same states as $SU(6) \otimes O(3)$ based on $\rho = +$ Dirac spiors
- Patterns of lattice baryonic states are similar to patterns of physical resonance states.
- Lots of lattice states; no signs of chiral restoration

The path forward

- No multiparticle states have been identified so far using three-quark operators
- Multiparticle operators (e.g, πN, $\pi\pi N$) must be added to realize significant couplings of three-quark states and their decay products.
- Moving operators and larger volumes will allow determination of elastic phase shifts using Luscher's formalism.
- Much remains to be learned as m_π is lowered toward the physical limit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E (MeV)
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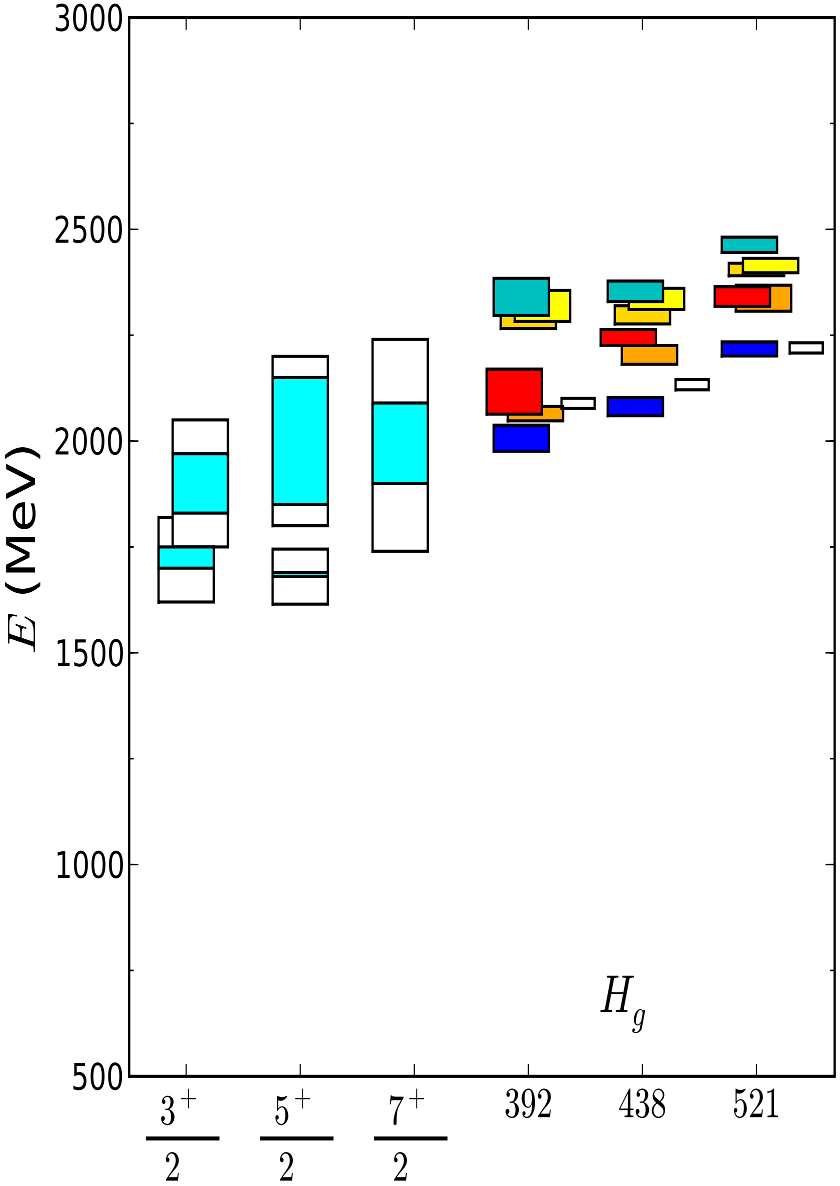
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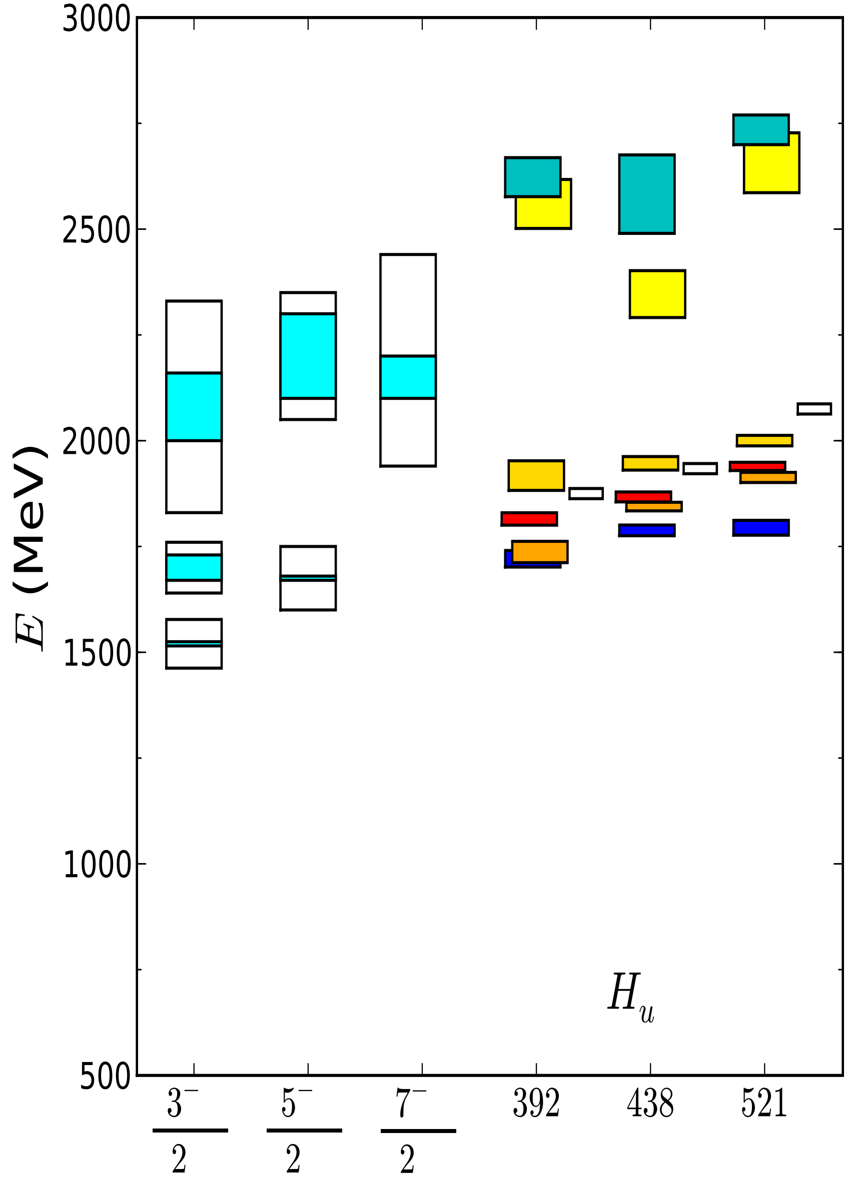
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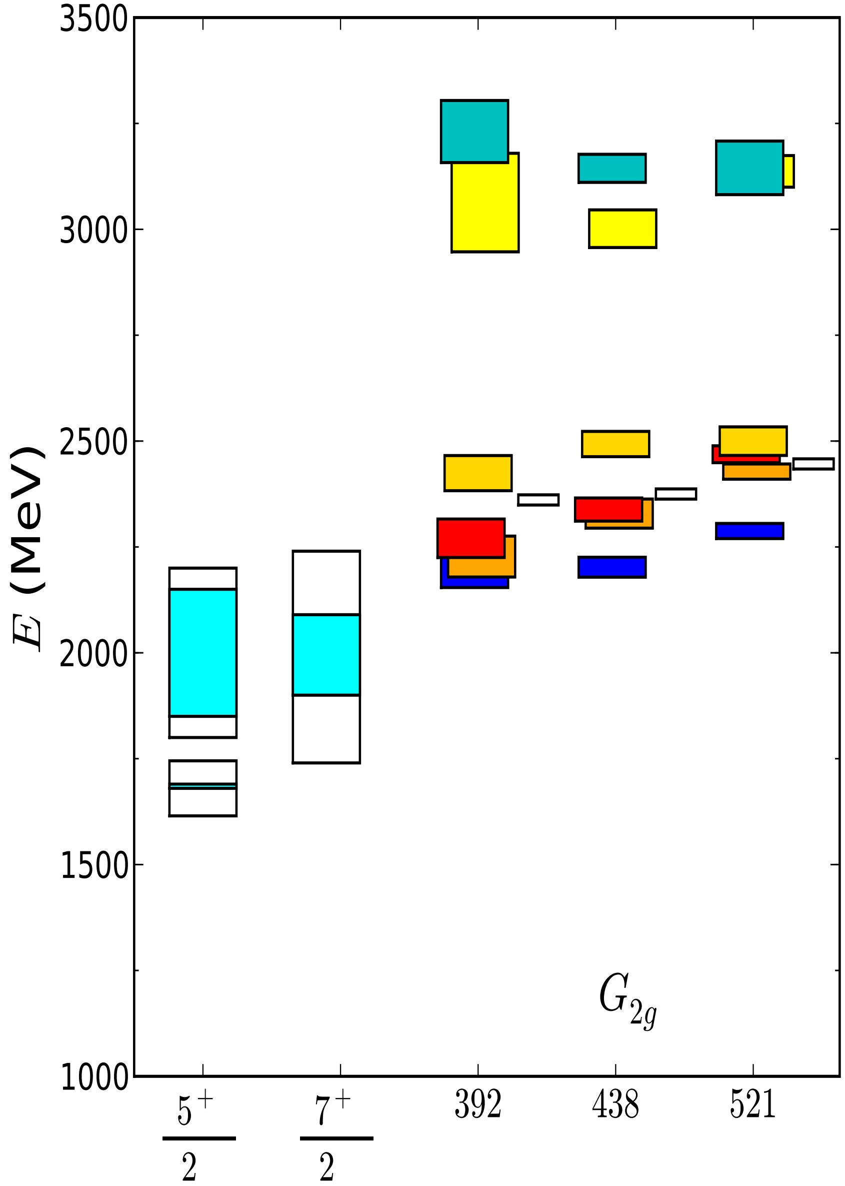
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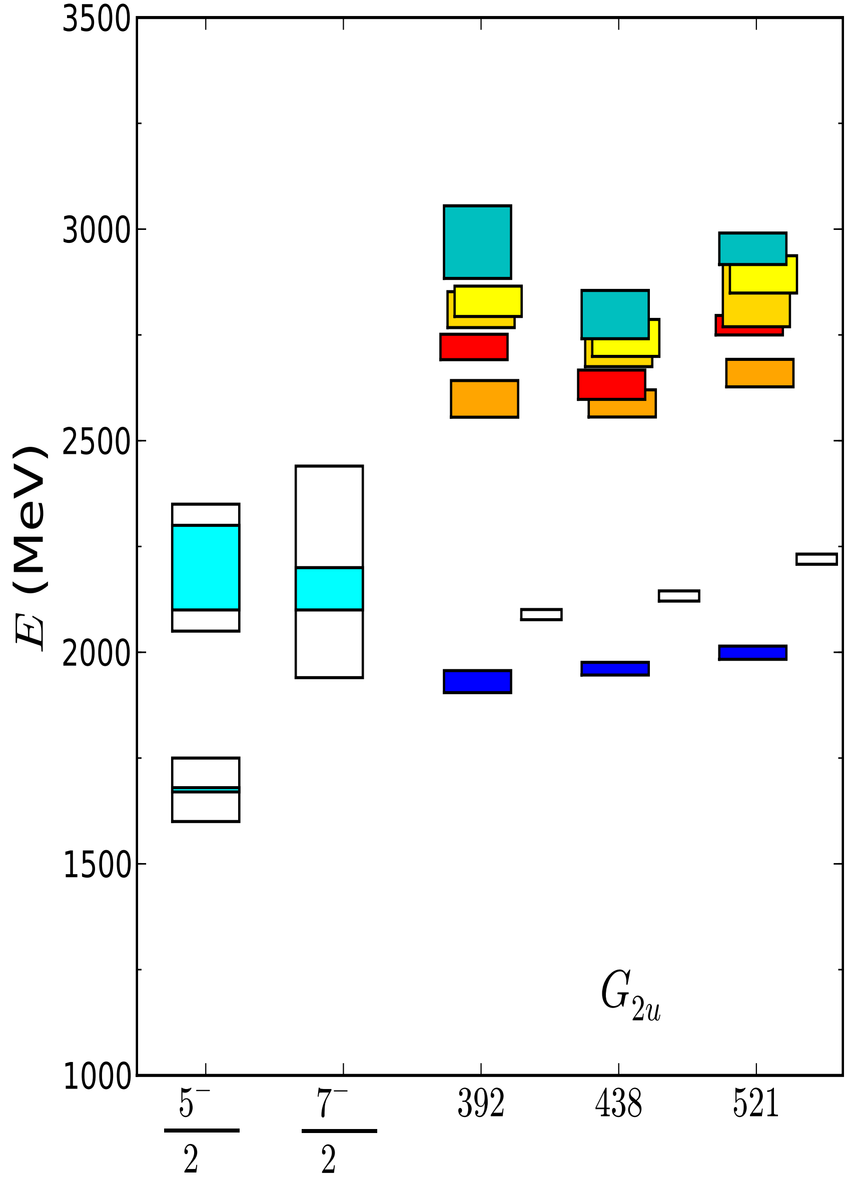
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